

# EMBED ZEE NEUTRINO MASS MODEL INTO SUSY

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In this talk, I summarize a work done in collaboration <sup>1</sup> with Otto Kong on the Zee neutrino mass model. We show that the MSSM with explicit  $R$ -parity violation actually contains the Zee model with the right-handed sleptons as the Zee singlet. We determine the conditions on the parameter space such that the neutrino mass matrix provides a viable texture that explains the atmospheric and solar data.

## 1 Introduction

We have seen substantial amount of experimental evidences from solar and atmospheric neutrino experiments that neutrinos in fact have masses. Among the experiments, SuperKamiokande <sup>2</sup> provided the strongest evidence for the atmospheric neutrino deficit, especially the impressive zenith angle distribution. The neutrino oscillation of  $\nu_\mu \rightarrow \nu_\tau$  provides the best explanation for the atmospheric neutrino deficit. On the other hand, the solar neutrino deficit is best explained by  $\nu_e \rightarrow \nu_\mu, \nu_\tau$ .

So where do we stand if neutrinos do in fact oscillate?

1. Neutrino oscillation necessarily implies neutrinos have masses and of different masses.
2. However, we do not know the absolute values of the masses. We only know the mass differences. The mass difference required to explain the atmospheric neutrino deficit is <sup>3</sup>

$$\Delta m_{\text{atm}}^2 \sim 3 \cdot 10^{-3} \text{ eV}^2 ,$$

while a few solutions to the solar neutrino deficit exist. For example, the LMA solution requires a mass difference of <sup>3</sup>

$$\Delta m_{\text{solar}}^2 \sim 10^{-5} \text{ eV}^2 \quad (\text{MSW}) .$$

3. Though we do not know the absolute mass scale of the neutrinos, we have indirect constraints from various sources. The cosmological constraint

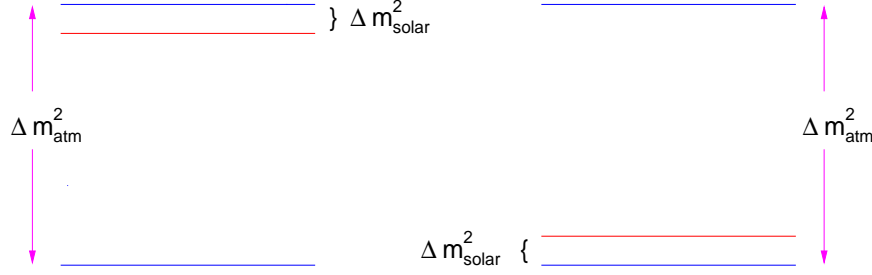
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$\Omega_{\text{hot}} \lesssim 0.1$  implies  $m_\nu \lesssim 3$  eV, assuming neutrinos make up the hot dark matter. The end point of Tritium decay also constrains  $m_{\nu_e} \lesssim 2.2$  eV. Nevertheless, the best constraint comes from the neutrinoless double beta ( $0\nu\beta\beta$ ) decay. The absence of  $0\nu\beta\beta$  decay put an upper bound on the effective neutrino mass, as

$$\langle m_\nu \rangle_e \equiv \sum_i m_{\nu_i} V_{ei}^2 \lesssim 0.2 \text{ eV}.$$

We know of two widely separated mass scales in neutrinos:  $\Delta m_{\text{atm}}^2$  and  $\Delta m_{\text{solar}}^2$ . Two possibilities of arranging the three neutrino masses exist: (1)  $m_1 \ll m_2 \sim m_3$  or (2)  $m_1 \sim m_2 \ll m_3$ , assuming  $m_1 < m_2 < m_3$ :



## 2 Types of neutrino mass

There are three types of neutrino mass according to the structure of the mass term.

(i) Dirac neutrino mass:  $\overline{\psi_L} \mathcal{M}_D \chi_R + h.c.$ , in which  $\chi_R$  is the right-handed neutrino field. This is analogous to the Dirac mass term for charged leptons. However, this term is not allowed in the SM, because the bare mass term is forbidden by gauge invariance and the SM does not have the right-handed neutrino field. Even in the case of charged leptons, the Dirac mass term must be derived from the Yukawa term with a Higgs field or equivalent, in order that gauge invariance is fulfilled before the symmetry breaking, followed by spontaneous symmetry breaking that the Higgs field develops a VEV.

(ii) Left-handed majorana neutrino mass:  $\psi_L^T C^{-1} \mathcal{M}_L \psi_L$ , where  $C$  is the charge conjugation operator. Again, this bare mass term is not allowed in the SM due to gauge invariance. Therefore, it must be derived from a Yukawa term with a Higgs field or equivalent. However, in this case a  $I = 1, Y = 2$  Higgs field is required to generate such a mass term. SM does not have such a Higgs field.

(iii) Right-handed marjorana mass:  $\chi_R^T C^{-1} \mathcal{M}_R \chi_L$ . In the SM, there is no right-handed neutrino field.

Therefore, to generate nonzero neutrino mass one has to include new physics beyond the SM. In both (i) and (iii) a right-handed field has to be introduced while the case (ii) does not necessarily require a right-handed field.

The hierarchy between the small neutrino mass and the charged lepton mass tells us something special about the mechanism that generates the neutrino mass, otherwise a fine tuning of the small Yukawa coupling for neutrinos is needed. A natural way to generate small neutrino mass is the see-saw mechanism, making use of a very large mass scale. Suppose there exist heavy right-handed neutrino fields  $\chi_R$ 's that couple to the left-handed neutrino fields via the usual Yukawa coupling. After electroweak symmetry breaking,

$$\mathcal{L} = \overline{\nu_{L_i}} (\mathcal{M}_D)_{ij} \chi_{R_j} + \chi_{R_i}^T C^{-1} (\mathcal{M}_R)_{ij} \chi_{R_j} + h.c. , \quad (1)$$

where the first term is the Dirac mass term for the neutrinos and the last term is the majorana mass for the right-handed fields. We can then write the mass matrix as

$$\frac{1}{2} \begin{pmatrix} \overline{\nu_L} & \overline{\chi_R^{(c)}} \end{pmatrix} \begin{pmatrix} \mathcal{M}_L = 0 & \mathcal{M}_D \\ \mathcal{M}_D^T & \mathcal{M}_R \end{pmatrix} \begin{pmatrix} \nu_L^{(c)} \\ \chi_R \end{pmatrix} + h.c. \quad (2)$$

After diagonalizing the mass matrix, the mass matrix of the light neutrinos is given by

$$M_\nu = -\mathcal{M}_D \mathcal{M}_R^{-1} \mathcal{M}_D^T , \quad (3)$$

where  $\mathcal{M}_R^{-1}$  is the inverse of the majorana mass matrix. If  $\mathcal{M}_R$  is sufficiently large, it naturally obtains small neutrino mass. Or equivalently, in terms of a dim-5 operator:

$$\mathcal{L} = \frac{y_{ij}}{M_R} (L_i H_2)(L_j H_2) .$$

To explain the observed neutrino mass the scale of  $\mathcal{M}_R \sim 10^{10-13}$  GeV for a typical Yukawa coupling. Such an intermediate scale arouses a lot of theoretical speculations and interests. Should the  $\chi_R$  related to SUSY breaking or early unification (a prediction of the Type I string theory is that the string scale is around  $10^{11}$  GeV.)

Another natural way to generate small neutrino masses is to make use of loop suppression. This need not introduce right-handed neutrino fields, though new physics is still needed to generate the neutrino mass. One nice example is the Zee model <sup>4</sup>.

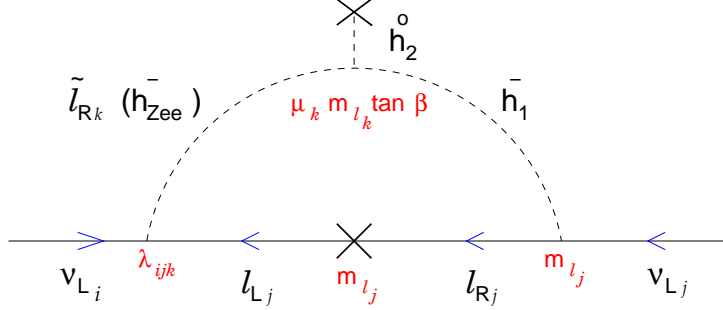


Figure 1. A Feynman diagram for the Zee model, embedded in the RPV SUSY framework.

### 3 Zee mass model

Zee model<sup>4</sup> provides an economical way to generate small neutrino masses with a favorable texture<sup>4,5,6</sup>. The model consists of a charged gauge singlet scalar  $h^-$ , which couples to lepton doublets  $\psi_{Lj}$  via the interaction

$$f^{ij} (\psi_{Li}^\alpha \mathcal{C} \psi_{Lj}^\beta) \epsilon_{\alpha\beta} h^- , \quad (4)$$

where  $\alpha, \beta$  are the SU(2) indices,  $i, j$  are the generation indices,  $\mathcal{C}$  is the charge-conjugation matrix, and  $f^{ij}$  are Yukawa couplings antisymmetric in  $i$  and  $j$ . Another ingredient of the model is an extra Higgs doublet (in addition to the one that gives masses to charged leptons) that develops a VEV and thus provides mixing between the charged Higgs boson and the Zee singlet. The one-loop diagram for the Zee model is depicted in Fig. 1.

The Zee model can provide a mass matrix of the following texture<sup>5,6</sup>

$$\begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & 0 & \epsilon \\ m_{e\tau} & \epsilon & 0 \end{pmatrix} , \quad (5)$$

where  $\epsilon$  is small compared with  $m_{e\mu}$  and  $m_{e\tau}$ , which is able to provide a compatible mass pattern that explains the atmospheric and solar neutrino data. Diagonal elements are guaranteed to vanish while the  $m_{\mu\tau}$  entry, denoted by  $\epsilon$ , has to be suppressed by some means. Moreover,  $m_{e\mu} \sim m_{e\tau}$  is required to give the maximal mixing solution for the atmospheric neutrinos.

First, take  $\epsilon = 0$  the matrix can be diagonalized by

$$\begin{pmatrix} \nu_{Le} \\ \nu_{L\mu} \\ \nu_{L\tau} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{m_{e\mu}}{\sqrt{2}m} & \frac{-m_{e\mu}}{\sqrt{2}m} & \frac{-m_{e\tau}}{m} \\ \frac{m_{e\tau}}{\sqrt{2}m} & \frac{-m_{e\tau}}{\sqrt{2}m} & \frac{m_{e\mu}}{m} \end{pmatrix} \begin{pmatrix} \nu_{L1} \\ \nu_{L2} \\ \nu_{L3} \end{pmatrix}, \quad (6)$$

with the eigenvalues  $m, -m, 0$  for  $\nu_{L1}, \nu_{L2}, \nu_{L3}$ , respectively, and  $m = \sqrt{m_{e\mu}^2 + m_{e\tau}^2}$ . The atmospheric mass-squared difference  $\Delta m_{\text{atm}}^2 \simeq 3 \times 10^{-3} \text{eV}^2$ , is to be identified with  $m^2 = m_{e\mu}^2 + m_{e\tau}^2$ . The transition probabilities for  $\nu_{L\mu}$  are

$$P_{\nu_{L\mu} \rightarrow \nu_{Le}} = 0,$$

$$P_{\nu_{L\mu} \rightarrow \nu_{L\tau}} = 4 \left( \frac{m_{e\mu} m_{e\tau}}{m_{e\mu}^2 + m_{e\tau}^2} \right)^2 \sin^2 \left( \frac{(m_{e\mu}^2 + m_{e\tau}^2)L}{4E} \right).$$

If  $m_{e\mu} \simeq m_{e\tau}$ , then  $\sin^2 2\theta_{\text{atm}} \simeq 1$ . This provides the maximal mixing solution for the atmospheric neutrino anomaly.

If we choose a nonzero  $\epsilon$ , but keep  $\epsilon \ll m_{e\mu, e\tau}$ . Then after diagonalizing the matrix we have the following eigenvalues

$$m_{\nu 1} = \sqrt{m_{e\mu}^2 + m_{e\tau}^2} + \epsilon \frac{m_{e\mu} m_{e\tau}}{m_{e\mu}^2 + m_{e\tau}^2},$$

$$m_{\nu 2} = -\sqrt{m_{e\mu}^2 + m_{e\tau}^2} + \epsilon \frac{m_{e\mu} m_{e\tau}}{m_{e\mu}^2 + m_{e\tau}^2},$$

$$m_{\nu 3} = -2\epsilon \frac{m_{e\mu} m_{e\tau}}{m_{e\mu}^2 + m_{e\tau}^2}.$$

The mass-square difference between  $m_{\nu 1}^2$  and  $m_{\nu 2}^2$  can be fitted to the solar neutrino mass. If one takes the LMA solution and requires

$$4\epsilon \frac{m_{e\mu} m_{e\tau}}{\sqrt{m_{e\mu}^2 + m_{e\tau}^2}} = \Delta m_{\text{sol}}^2 \simeq 2 \times 10^{-5} \text{eV}^2,$$

giving (we have used  $m_{e\mu} \simeq m_{e\tau}$ )

$$\frac{\epsilon}{m_{e\mu}} \sim 5 \times 10^{-3}.$$

#### 4 Neutrino mass in SUSY

The original Zee model was not embedded into any grand unified theories or supersymmetric models. It would be very interesting if the Zee model

naturally exists in some GUT or SUSY theories. In fact, the minimal supersymmetric standard model (MSSM) with a minimal extension, namely, the  $R$ -parity violation, contains the Zee model. The right-handed sleptons in SUSY have the right quantum numbers to play the role of the charged Zee singlet. The  $R$ -parity-violating (RPV)  $\lambda$ -type couplings could provide the terms in Eq.(4). It is also easy to see that the RPV bilinear  $\mu$ -type couplings ( $\mu_i LH_2$ ) would allow the second Higgs doublet  $H_2$  in SUSY to be the second ingredient of the Zee model.

However, in RPV SUSY framework, there are three other sources for neutrino masses, in addition to the Zee model contribution. They are (i) the tree-level mixing with the higgsinos and gauginos, (ii) the one-loop diagram that involves the usual mass mixing between the left-handed and right-handed sleptons proportional to  $m_\ell (A_\ell^E - \mu \tan \beta)$ , and (iii) the one-loop diagram that again involves the mixing between the left-handed and right-handed sleptons but this time via the  $\lambda$  and  $\mu_i$  couplings. They may deviate from the texture of the Zee mass matrix of Eq. (5).

The *tree-level mixing* among the higgsinos, gauginos, and neutrinos gives rise to a  $7 \times 7$  neutral fermion mass matrix  $\mathcal{M}_\mathcal{N}$  under SVP <sup>7</sup>:

$$\mathcal{M}_\mathcal{N} = \left( \begin{array}{cccc|ccc} M_1 & 0 & g'v_2/2 & -g'v_1/2 & 0 & 0 & 0 \\ 0 & M_2 & -gv_2/2 & gv_1/2 & 0 & 0 & 0 \\ g'v_2/2 & -gv_2/2 & 0 & -\mu & -\mu_1 & -\mu_2 & -\mu_3 \\ -g'v_1/2 & gv_1/2 & -\mu & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & -\mu_1 & 0 & (m_\nu^0)_{11} & (m_\nu^0)_{12} & (m_\nu^0)_{13} \\ 0 & 0 & -\mu_2 & 0 & (m_\nu^0)_{21} & (m_\nu^0)_{22} & (m_\nu^0)_{23} \\ 0 & 0 & -\mu_3 & 0 & (m_\nu^0)_{31} & (m_\nu^0)_{32} & (m_\nu^0)_{33} \end{array} \right), \quad (7)$$

whose basis is  $(-i\tilde{B}, -i\tilde{W}, \tilde{h}_2^0, \tilde{h}_1^0, \nu_{Le}, \nu_{L\mu}, \nu_{L\tau})$ .

In the above  $7 \times 7$  matrix, the whole lower-right  $3 \times 3$  block  $(m_\nu^0)$  is zero at tree level. They are induced via one-loop contributions. We can write the mass matrix in the form of block submatrices:

$$\mathcal{M}_\mathcal{N} = \left( \begin{array}{c|c} \mathcal{M} & \xi^T \\ \hline \xi & m_\nu^0 \end{array} \right), \quad (8)$$

where  $\mathcal{M}$  is the upper-left  $4 \times 4$  neutralino mass matrix,  $\xi$  is the  $3 \times 4$  block, and  $m_\nu^0$  is the lower-right  $3 \times 3$  neutrino block in the  $7 \times 7$  matrix. The resulting neutrino mass matrix after block diagonalization is given by

$$(m_\nu) = -\xi \mathcal{M}^{-1} \xi^T + (m_\nu^0). \quad (9)$$

The first term here corresponds to the tree level contributions, which are see-saw suppressed.

Through this gaugino-higgsino mixing, nonzero  $\mu_i$ 's give tree-level see-saw type contributions to  $(m_\nu)_{ij}$  proportional to  $\mu_i\mu_j$ , given by

$$(m_\nu)_{ij} = -\frac{v^2 \cos^2 \beta (g^2 M_1 + g'^2 M_2)}{2\mu [2\mu M_1 M_2 - v^2 \sin \beta \cos \beta (g^2 M_1 + g'^2 M_2)]} \mu_i \mu_j. \quad (10)$$

A diagonal  $(m_\nu)_{kk}$  term is always present for a nonzero  $\mu_k$ . To eliminate these tree-level terms requires either very stringent constraints on the parameter space or extra Higgs superfields beyond the MSSM spectrum. This is a major difficulty of the present MSSM formulation of supersymmetric Zee model.

*Zee mechanism.* The Feynman diagram is shown in Fig. 1. The  $\tilde{\ell}_{R_k}$  is the charged Zee singlet. To complete the diagram the charged Higgs boson  $h_1^-$  from the Higgs doublet  $H_1$  is on the other side of the loop and a  $\tilde{\ell}_{R_k}$ - $h_1^-$  mixing at the top of the loop is provided by a  $F$  term of  $L_k$ :  $\mu_k m_{\ell_k} h_1^- \tilde{\ell}_{R_k}^* \langle h_2^0 \rangle / \langle h_1^0 \rangle$ , where  $h_2^0$  takes on its VEV, for a nonzero  $\mu_k$ . Thus, the neutrino mass term  $(m_\nu^0)_{ij}$  has a

$$\mu_k m_{\ell_k} \lambda_{ijk} (m_{\ell_j}^2 - m_{\ell_i}^2) \quad (11)$$

dependence, where  $m_{\ell_i}$ 's are the charged lepton masses.

*LR slepton mass mixing* comes from the one-loop diagram with two  $\lambda$ -coupling vertices and the usual  $(A^E - \mu \tan \beta)$ -type  $LR$  slepton mixing. Neglecting the off-diagonal entries in  $A^E$ , the contribution to  $(m_\nu^0)_{ij}$  with the pair  $\lambda_{ilk}$  and  $\lambda_{jkl}$  is proportional to

$$[ (A_k^E - \mu \tan \beta) + (1 - \delta_{kl})(A_l^E - \mu \tan \beta) ] m_{\ell_k} m_{\ell_l} \lambda_{ilk} \lambda_{jkl}. \quad (12)$$

*LR slepton mass mixing via RPV couplings* comes from a  $F$  term of  $L_i$ :  $\mu_i \lambda_{ijk} \tilde{\ell}_{L_j} \tilde{\ell}_{R_k}^* \langle h_2^0 \rangle$ , where  $h_2^0$  takes on the VEV. This is similar to the  $\tilde{\ell}_{R_k}$ - $h_1^-$  mixing in the Zee model, except that this time we have a  $\lambda$ -type coupling in place of the Yukawa coupling. With a specific choice of a set of nonzero  $\mu_i$ 's and  $\lambda$ 's, this type of mixing gives rise to the off-diagonal  $(m_\nu^0)_{ij}$  terms only and, therefore, of particular interest to our perspectives of Zee model. Taking the pair  $\lambda_{ilk}$  and  $\lambda_{jhl}$  for the fermion vertices and a  $F$  term of  $L_g$  providing a coupling for the scalar vertex in the presence of a  $\mu_g$  and a  $\lambda_{ghk}$ , a  $(m_\nu^0)_{ij}$  term is generated and proportional to

$$\mu_g m_{\ell_i} \lambda_{ghk} \lambda_{ilk} \lambda_{jhl}. \quad (13)$$

When we allow only one nonzero  $\lambda$  at a time, the only contribution comes from  $\lambda_{ijj}$  but not from those with distinct indices. Suppose we have nonzero  $\lambda_{ijj}$  and  $\mu_j$ , there is a contribution to the off-diagonal  $(m_\nu^0)_{ij}$  with a  $\mu_j m_{\ell_j} \lambda_{ijj}^3$  dependence.

We conclude that a minimal set of RPV couplings needed to give the zeroth order Zee matrix is

$$\{ \lambda_{12k}, \lambda_{13k}, \mu_k \}.$$

As at least one of the two  $\lambda$ 's has the form  $\lambda_{ikk}$  ( $\equiv -\lambda_{kik}$ ), all types of contributions that have been discussed above are there. We want to make the contribution from the Zee mechanism dominate over the others, or at least to suppress the diagonal entries in  $(m_\nu)$ . This necessarily requires suppression of the contributions from the tree-level see-saw mechanism and from the  $(A^E - \mu \tan\beta)$ -type  $LR$  slepton mixing. So, it is the Zee mechanism and the  $LR$  mixing via RPV couplings are required to be the dominant ones.

## 5 Scenarios and conditions to maintain Zee Texture

Because of space limitation we only show the best scenario:  $\{\lambda_{123}, \lambda_{133}, \text{ and } \mu_3\}$ . The resulting neutrino mass matrix is given by

$$\begin{pmatrix} C'_4 m_\tau^2 \lambda_{133}^2 & C'_2 m_\tau m_\mu^2 \mu_3 \lambda_{123} + C_5 m_\tau \mu_3 \lambda_{123} \lambda_{133}^2 & C'_2 m_\tau^3 \mu_3 \lambda_{133} + C_5 m_\tau \mu_3 \lambda_{133}^3 \\ 0 & 0 & 0 \\ C_1 \mu_3^2 & & \end{pmatrix} \quad (14)$$

where

$$\begin{aligned} C_1 &= -\frac{v^2 \cos^2\beta (g^2 M_1 + g'^2 M_2)}{2\mu [2\mu M_1 M_2 - v^2 \sin\beta \cos\beta (g^2 M_1 + g'^2 M_2)]}, \\ C'_4 &= -\frac{1}{16\pi^2} (A_\tau^E - \mu \tan\beta) f(M_{\tilde{\tau}_L}^2, M_{\tilde{\tau}_R}^2), \\ C'_2 &= \frac{-1}{16\pi^2} \frac{\sqrt{2} \tan\beta}{v \cos\beta} f(M_{h_1^-}^2, M_{\tilde{\tau}_R}^2), \\ C_5 &= -\frac{1}{16\pi^2} \frac{v \sin\beta}{\sqrt{2}} f(M_{e_L}^2, M_{\tilde{\tau}_R}^2), \end{aligned} \quad (15)$$

where  $f(x, y) = \frac{1}{x-y} \log(y/x)$ .

In the above, we have neglected terms suppressed by  $m_e/m_\mu$  or  $m_e/m_\tau$ . In order to maintain the zeroth order Zee texture, we need  $m_{e\mu}$  and  $m_{e\tau}$  to dominate over the other entries. Moreover, we need  $m_{e\mu} \sim m_{e\tau} \sim \sqrt{\Delta M_{\text{atm}}^2} (\sim 5 \times 10^{-11} \text{ GeV})$ .

Requiring the tree-level gaugino-higgsino mixing contribution to be well below  $m_{e\mu}$  gives

$$\mu_3^2 \cos^2\beta \ll \mu^2 M_1 (1 \times 10^{-14} \text{ GeV}^{-1}). \quad (16)$$



For the  $(A_k^E - \mu \tan \beta)$   $LR$  slepton mixing contribution to be much smaller than  $m_{e\mu}$ , we have

$$\lambda_{133}^2 \ll \frac{\text{Max}(M_{\tau_L}^2, M_{\tau_R}^2)}{(A_\tau^E - \mu \tan \beta)} (2.5 \times 10^{-9} \text{ GeV}^{-1}) . \quad (17)$$

This corresponds to  $m_{ee}$ . It tells us that  $\lambda_{133}$  can hardly be much larger than  $10^{-3}$ . On the other hand,  $\lambda_{123}$  is constrained differently because it does not contribute to this type of neutrino mass term.

From the tree-level Zee-scalar mediated  $\mu$  decay, the constraint is

$$\frac{\lambda_{123}^2}{M_{\tau_R}^2} \leq 10^{-8} \text{ GeV}^{-2} , \quad (18)$$

which tells us that  $\lambda_{123}$  can be as large as order of 0.01 for scalar masses of order of  $O(100)$  GeV.

Both  $m_{e\mu}$  and  $m_{e\tau}$  have two terms. Let us look at  $m_{e\mu}$  first. For the first term in  $m_{e\mu}$  (the one with a  $C_2'$  dependence) in Eq. (14) to give the required value of atmospheric neutrino mass, we need

$$m_{e\mu} \sim \frac{\mu_3 \lambda_{123}}{\cos^2 \beta} \frac{1}{\max(M_{h_1^-}^2, M_{\tau_R}^2)} (7 \times 10^{-7} \text{ GeV}^2) \sim (5 \times 10^{-11} \text{ GeV}) \quad (19)$$

or

$$(\mu_3 \cos \beta) \lambda_{123} \sim \cos^3 \beta \max(M_{h_1^-}^2, M_{\tau_R}^2) (7 \times 10^{-5} \text{ GeV}^{-1}) . \quad (20)$$

This result looks relatively promising. If we take  $\cos \beta = 0.02$ , all the involved scalar masses at 100 GeV and  $\lambda_{123}$  at the corresponding limiting 0.01 value,  $\mu_3 \cos \beta$  has to be at  $5.6 \times 10^{-4} \text{ GeV}$  to fit the requirement. This means pushing for larger  $M_1$  (and  $M_2$ ) and  $\mu$  values but may not be ruled out.

The corresponding first term in  $m_{e\tau}$  has a  $\lambda_{133}$  dependence in the place of  $\lambda_{123}$  with an extra enhancement of  $m_\tau^2/m_\mu^2$ , in comparison to  $m_{e\mu}$ . That is to say, requiring  $m_{e\mu} \approx m_{e\tau}$  gives, in this case,

$$\lambda_{133} \approx \frac{m_\mu^2}{m_\tau^2} \lambda_{123} . \quad (21)$$

This gives a small  $\lambda_{133}$  easily satisfying Eq. (17). The small  $\lambda_{133}$  also suppresses the second terms in both  $m_{e\mu}$  and  $m_{e\tau}$ , the  $C_5$  dependent terms in Eq. (14).

To produce the neutrino mass matrix beyond the zeroth order Zee texture, the subdominating first-order contributions are required to be substantially smaller in order to fit the solar neutrino data. Here, it is obvious that it is

difficult to further suppress the tree level gaugino-higgsino mixing contribution to  $m_{\tau\tau}$ , which makes it even more difficult to get the scenario to work. Explicitly, the requirement for the solar neutrino is

$$\mu_3^2 \cos^2 \beta \sim \mu^2 M_1 (1 \times 10^{-16} \text{ GeV}^{-1}) . \quad (22)$$

## 6 A general version of supersymmetric Zee model

The conditions for maintaining the Zee neutrino mass matrix texture is extremely stringent, if not impossible, mainly because of the tree-level mixings via the bilinear RPV couplings. An alternative without the bilinear RPV couplings is to introduce an additional pair of Higgs doublet superfields. Denoting them by  $H_3$  and  $H_4$ , bearing the same quantum numbers as  $H_1$  and  $H_2$ , respectively, RPV terms of the form

$$\epsilon_{\alpha\beta} \lambda_k^H H_1^\alpha H_3^\beta E_k^c$$

can be introduced. With a trivial extension of notations we obtain a Zee diagram contribution to  $(m_\nu)_{ij}$  through  $\lambda_{ijk}$  as follows :

$$\frac{-1}{16\pi^2} \frac{\langle h_3^0 \rangle}{\langle h_1^0 \rangle} (m_{\ell_j}^2 - m_{\ell_i}^2) \lambda_{ijk} \lambda_k^H A_k^H f(M_{h_1}^2, M_{\ell_{Rk}}^2) . \quad (23)$$

Here the slepton  $\tilde{\ell}_{Rk}$  keeps the role of the Zee singlet. Notice that the second Higgs doublet of the Zee model, corresponding to  $H_3$  here, is assumed not to have couplings of the form  $L_i H_3 E_j^c$ . The condition for the  $LR$  slepton mixing contribution to be below the required  $m_{e\mu}$  would be the same as discussed in the last section.

However, there is a new contribution to  $(m_\nu)_{kk}$  given by

$$\frac{-1}{16\pi^2} \frac{\langle h_3^0 \rangle^2}{\langle h_1^0 \rangle^2} m_{\ell_k}^2 (\lambda_k^H)^2 A_k^H f(M_{h_1}^2, M_{\ell_{Rk}}^2) . \quad (24)$$

This is a consequence of the fact that the term  $\lambda_k^H H_1^\alpha H_3^\beta E_k^c$  provides new mass mixings for the charged Higgsinos and the charged leptons. The essential difference here is that unlike the  $\mu_i$  terms the  $\lambda_k^H H_1^\alpha H_3^\beta E_k^c$  term does not contribute to the mixings between neutrinos and the gauginos and higgsinos on tree level.

Similar to the above we are interested in only the minimal set of couplings  $\{\lambda_{12k}, \lambda_{13k}, \lambda_k^H\}$  with a specific  $k$ . For expression (23) to give the right value to  $m_{e\mu}$ , we need

$$\lambda_{12k} \lambda_k^H \sim \frac{\text{Max}(M_{h_1}^2, M_{\ell_{Rk}}^2)}{A_k^H} \frac{\langle h_1^0 \rangle}{\langle h_3^0 \rangle} (7 \times 10^{-7} \text{ GeV}^{-1}) , \quad (25)$$

and similarly for  $m_{e\tau}$ , it requires  $\lambda_{13k} = (m_\mu^2/m_\tau^2)\lambda_{12k}$ . This condition is easy to satisfy when we take  $\langle h_3^0 \rangle / \langle h_1^0 \rangle = 0.1$ . For Eq. (23) to dominate over Eq. (24), it requires

$$\lambda_{12k} \gg \lambda_k^H \frac{\langle h_3^0 \rangle}{\langle h_1^0 \rangle} \frac{m_{\ell_k}^2}{m_\mu^2}, \quad \lambda_{13k} \gg \lambda_k^H \frac{\langle h_3^0 \rangle}{\langle h_1^0 \rangle} \frac{m_{\ell_k}^2}{m_\tau^2}. \quad (26)$$

The most favorable scenario is then the  $k = 1$  case, where  $m_{\ell_k}$  is just the  $m_e$ . The above requirements are then easily satisfied. Also, the requirement for suppression of the  $LR$  slepton mixing is the same as before, and we also have Eq. (18) from the tree-level Zee-scalar induced muon decay. All these constraints can now be easily satisfied. Hence, such a supersymmetric Zee model looks very feasible.

## 7 Conclusions

Zee model provides a viable texture that explains the data. The minimal extension of MSSM with  $R$ -parity violation actually contains the Zee model, with the right-handed sleptons  $\tilde{\ell}_R$  as the charged singlet,  $\lambda_{ijk}$  couplings providing lepton-number violation, and  $H_u$  providing the mixing.

However, there are other sources of neutrino mass in RPV SUSY, some of which wipe away the favorable texture. In order for the Zee contribution to dominate over the others we pick the best minimal scheme  $\{\lambda_{12k}, \lambda_{13k}, \mu_k\}$ ,  $k = 3$ , and determine the requirements on the parameter space, which turns out quite stringent but still possible.

Finally, we offered a further consideration that abandons the bilinear RPV couplings but introduces two additional Higgs doublets. This model turns out quite feasible.

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